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Latent Assets

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IT IS NOW A commonplace that corporate insiders possess an informational advantage over investors at large. On the one hand, the imperfect observability of the actions of corporate managements and of corporate investment opportunities has given rise to a now extensive literature on the problems of agency and adverse selection that arise in the corporate context. On the other, since Ross (1972) there has been considerable interest in the possibility of signaling resolutions to the problems arising from the superior information of management about corporate prospects. Nevertheless, it seems that signaling can provide only a partial resolution of the information asymmetry, and in this paper I want to consider some implications of the fact that it is costly for investors at large to determine the value of the assets held by the corporation. At the risk of oversimplification I shall distinguish between those assets whose values are known and agreed on by investors and reflected in share prices and those assets whose values are not reflected in share prices; the latter I shall refer to as latent assets. Investor rationality requires that the expected value of latent assets for any given firm be equal to zero, so that latent assets may be negative as well as positive.

Systematic evidence that share prices do not always fully reflect the value of assets held for corporations is hard to come by, not least because of the tenuous relation between our valuation models for most assets and observable variables. Nevertheless, there is considerable anecdotal and indirect evidence to this effect. For example, the substantial bid premia that are observed in the current wave of takeovers constitute prima facie evidence of the existence of latent assets, even though the premia themselves may be made possible by tax and operating efficiencies and wealth redistributions rather than the exploration of undervaluation. Thus a bid premium of 30% of the pre-bid market price can be consistent with the market knowing that the value of the assets in their alternative use is 30% higher than in their current use, only if the market assigns a zero probability to the emergence of the bid—that seems unreasonable. If the market assigns only a 20% probability to the bid, then a 30% bid premium implies that the assets are worth 49% more in the alternative than in the current use—but then a 20% probability seems implausibly low, and a 50% probability implies an 86% differential.[1] If investors can be so ignorant of the value of assets in alternative

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[1] Let A denote the value of assets in current use and V their value to a bidder. If M is the current market value of the firm then $M = pV + (1 - p)A$, where p is the probability of a succesful bid. The example in the text assumes $V = 1.3M$.

uses it is reasonable to assume that they have less than perfect information about their values in current use. Further evidence to this effect lies in stock price reactions to many corporate announcements.

I shall consider the implications of latent assets from two perspectives. First I shall take an extreme case in which investors are unable to observe the value of the firm's assets at any cost, and consider some implications of this for corporate policy. In doing so I shall offer an explanation for a problem that has puzzled me for some time—namely the fact that gold mines are widely exploited despite the fact that standard present value analyses generally imply that it is optimal to defer exploitation indefinitely. Generalizing from the example of the gold mine, I shall argue that the latency of asset values induces firms to convert them into cash when they are positive, even when it is costly to do so—and conversely, to postpone realization when they are negative, even when that is costly. Among other things, our results point to the dangers of making inferences from stock price reactions to the announcement of corporate decisions, without giving due attention to the objective of the management.

Secondly, I shall consider the other extreme case in which corporate policies are fixed, but investors are able to acquire information about the value of latent assets at a cost. The main point I shall stress here is the interdependence of individual information acquisition decisions. In a market populated by small investors, the advantage to any individual of acquiring information about the latent assets of a firm may be very small if those assets have a low current yield and no other investor acquires the same information. On the other hand, if many other investors acquire the same information the share price will adjust to reflect it, rewarding those who acquired the information first. Thus, while too much information acquisition activity will compete away the rewards to acquiring information as in the classical efficient market, too little information acquisition activity also may make it not worthwhile for any individual to undertake it. What is true about information acquisition is also true about modes of analysis. If the market follows a demonstrably inappropriate procedure in valuing a particular asset, the rewards to having the correct procedure may yet be small, unless others can be persuaded of the superiority of your approach, and this may be difficult since, by definition, it does not correspond to market prices as well as the conventional procedure. Pity the man who alone knows how to value gold mines, for his reward shall be slight. Thus, I argue, market prices may reflect a large element of convention without there existing a strong tendency for this to be corrected. Of course there are some corrective forces. First among these is the activity of corporations themselves which have so far held fixed. To the extent that assets remain latent or under-valued by the market, there will be an incentive for managements to convert them to cash or run the risk of being acquired by someone else who will convert them. Thus the market for corporate control exerts a corrective force and even tends to increase the incentives for private information acquisition by hastening the day when the rewards of such activity will be enjoyed. Secondly, there are obviously benefits to our atomistic investors of coordinating their information gathering activities to ensure that when latent assets are discovered there is sufficient investment activity that the asset values come to be reflected in market prices. Strict coordination of investment, as in the mutual fund form, will not do the trick, for then the group claiming the information has an obvious incentive to misrepresent, since it will also have taken a position in the securities. However, a "disinterested" third party which is paid only for information acquisition, and gains a reputation for accuracy, may be able to provide the degree of coordination across investors necessary for its information to be quickly and profitably reflected in security prices. Brokerage house analysts provide a possible example.

I. The Gold Mine

My thinking about latent assets was stimulated by a puzzle that arose some years ago in developing with Eduardo Schwartz a model for valuing mineral resources.[2] A distinguishing feature of these assets is the highly unpredictable nature of the price of the extracted resource which, we argued, makes application of traditional valuation techniques extremely difficult. At the same time, this unpredictability provides the impetus for the development of futures markets, and these in turn provide information that can be used to infer the risk adjusted expected price change of the extracted mineral. Although we have used this model quite successfully, it had one embarrassing feature when applied to gold—it was very difficult to find parameter values that would make it optimal to extract the gold at all.

The reason for this apparent perversity is not hard to find. Stocks of gold in the world far exceed commercial inventory needs, and the bulk of the gold is held by individuals and central banks for investment purposes. This means that gold is an asset like a share of common stock, except that it does not pay a dividend.[3] Therefore, the present value of an ounce of gold deliverable at any time in the future is equal to the current spot price. However, it takes resources to extract the gold from the mine; and, assuming that the cost of extraction is rising at a rate less than the rate of interest, it will always pay to defer extraction. So our advice to owners of gold mines was always the same: "Wait!"[4] This makes perfectly good sense when it is just as good to own shares in a gold mine as it is to own the bullion directly.

Nevertheless, we observe that in practice most firms do extract the gold. In an opportunistic compromise between the descriptive and the prescriptive we added a knob to our model that could make this the optimal policy: we allowed for the possibility that the mine could be expropriated without compensation.[5] This

[2] See Brennan and Schwartz (1985).

[3] Nor does it cost much to store. In what follows we shall neglect storage costs. Other commodities that are held for commercial purposes are said to yield "convenience services" which correspond to the dividend on a share of common stock. See Brennan (1958).

[4] Keynes (1936) also though it was a (social) mistake to extract the gold: "··· the form of digging holes in the ground known as gold mining, which not only adds nothing whatever to the real wealth of the world, but involves the disutility of labour ···"

When I offered this same advice to a friend who was promoting a gold mine he thought it so laughable that he used to repeat it as evidence of the folly into which one could be led by excessive ratiocination.

[5] Our position was similar to that of Coleridge who one remarked to his dinner partner, "Madam, I accept your conclusion, but you must allow me to provide the logic for it."

means that the mine itself is no longer such a good store of value as the bullion itself. There are other considerations that may also serve to induce immediate extraction; for example, a government which is anxious to get its hands on the tax and royalty revenues from the mine may award a lease which will lapse if the mine is not developed. Rather than pursue these possibilities I shall focus on the asymmetry of the information between corporate insiders and investors about the value of the resource.

Asymmetry of information about the true value of a firm's assets seems particularly likely when those assets consist of a mineral resource, for not only is it difficult to communicate the raw information on which the valuation of the deposit depends and difficult to interpret those data, but there is also always the possibility that the information communicated is incomplete[6]: the market for unused gold mines suffers from much the same problems as the market for used cars. The information asymmetry will include such matters as amounts and grades of ore, costs of extraction and so on. In order to capture the information asymmetry in a simple fashion, we shall assume that it is common knowledge that the mine contains a unit of gold and that the gold can be extracted instantaneously, but that the cost of extraction is known only to management. We shall assume that the management of the mine is precluded from any signaling activities that would reveal its private information; in the appendix we consider a scenario in which the management can reveal its private information by precommitting to an output policy.[7]

Let g denote the current price of a unit of gold, which is also the value of a unit deliverable for sure at any time in the future, and let $M(s)$ denote the market value of the firm at time s. We assume that the future price path of the gold is known with certainty and that management maximizes a weighted average of present values of future stock prices:

$$\max \alpha \int_0^\infty e^{-(\alpha+r)s} M(s)\, ds, \tag{1}$$

where r is the interest rate and α is a parameter.

Let τ denote the date on which the gold is extracted from the mine. Since this cannot be signaled in advance, all firms with unexploited mines will be valued identically. Without loss of generality, we may write the value of such a firm as

$$M(s) = ge^{rs} - Q(s), \qquad s < \tau, \tag{2}$$

where $Q(s)$ is the present value at time s of the expected future costs of extraction

[6] See discussion by Grossman and Hart (1980). When one gold mining company was bid for, a commentator remarked: "It (Newmont Gold Mining Company) may also be a company of sizeable *hidden* value." *Los Angeles Times*, September 20, 1989 (emphasis added).

[7] Gold loans whose repayments are in bullion or bullion equivalent are a recent innovation which may function as precommitment devices. Such loans typically carry interest rates of 1.5–3% and are used to finance mine development. Note that while these loans provide a price hedge for the producer, this cannot be their sole purpose for such hedges are available more cheaply in the futures market. Since lenders send in their own analysts to evaluate a mine before making a loan, the size and yield on the loan also serve as signals of the quality of the mine. For a formal model of third party information production see Thakor (1982).

and ge^{rs} is the price of gold at time s, since under certainty the price will appreciate at the riskless rate. Note that since no firm will extract the gold if the cost of extraction exceeds the current gold price, it must be the case that $M(s) \geq 0$, or

$$Q(s) \leq ge^{rs}. \tag{3}$$

Extraction costs grow continuously over time at the known rate m. For ease of reference we shall refer to the extraction costs measured at time 0 as the real cost of extraction and denote it by C. Consider then a firm that extracts the gold at time τ. Its value at that time, $M^*(\tau)$, is equal to the difference between the value of the gold and the, now known, cost of extraction:

$$M^*(\tau) = ge^{r\tau} - Ce^{m\tau}. \tag{4}$$

Finally, for $s > \tau$, the true value of the firm is known so that

$$M(s) = M^*(\tau)e^{r(s-\tau)}. \tag{5}$$

Substituting for $M(s)$ and $M^*(s)$ from equations (2), (4) and (5) in (1), the objective function of the firm may be written as

$$\max_{\tau} V = g - \alpha \int_0^{\tau} Q(s)e^{-(\alpha+r)s}\, ds - Ce^{-(\alpha+r-m)\tau}. \tag{6}$$

The first and second order conditions for a maximum in (6) are

$$V' = -\alpha Q(\tau)e^{-(\alpha+r)\tau} + (\alpha + r - m)Ce^{-(\alpha+r-m)\tau} = 0, \tag{7}$$

$$V'' = [\alpha(\alpha + r)Q - \alpha Q' - (\alpha + r - m)^2 Ce^{m\tau}]e^{-(\alpha+r)\tau} < 0. \tag{8}$$

Solving the first order condition (7) for $C(t)$, the real cost of a mine for which it is optimal to produce at time t:

$$C(t) = \left(\frac{\alpha}{\alpha + r - m}\right)Q(t)e^{-mt}. \tag{9}$$

So

$$Q(0) = \left(\frac{\alpha + r - m}{\alpha}\right)C(0),$$

where $C(0)$ is the cost of the producing mine at time 0.

Then, substituting from (9) in (8), the second order condition may be written as

$$Q'/Q > m. \tag{10}$$

Under risk neutrality, equilibrium in the capital market requires that the expected rate of increase in the value of unexploited mines be equal to the risk-free interest rate. This expected rate of increase has two components—the rate of increase in the value of mines which remain unexploited, M', and the instantaneous capital gain or loss on a mine when it is brought into production, $(M^* - M)$, times the proportional rate at which new mines are brought into production. This rate is equal to the product of the hazard rate of the cost distribution function,[8] $h(C)$, and the time derivative of the real cost of the currently producing mine, $C'(t)$. Thus the condition for equilibrium in the capital market may be written as

$$(M^* - M)hC' + M' = Mr. \tag{11}$$

In order to complete the characterization of the equilibrium, it is necessary to specify the probability distribution function for the costs. For analytical convenience we take this as

$$F(C) = 1 - a/C^k, \qquad a > 0, \quad C > a^{1/k}. \tag{12}$$

This implies the hazard rate function $h(C) = k/C$. Then, computing M' from equation (2), and substituting for h and for M and M^* from (2) and (4) in (11) and using (9), we obtain

$$\eta(C'/C)Q - Q' + rQ = 0, \tag{13}$$

where $\eta \equiv k(r - m)/(\alpha + r - m) > 0$. But from (9) $C'/C = Q'/Q - m$, so that (13) may be written as

$$Q'/Q = (r - m\eta)/(1 - \eta) \equiv \gamma. \tag{14}$$

It may be verified that $\gamma > m$ as required by the second order condition (10) if $r > m$.[9]

The solution to equation (14) may be written as[10]

$$Q(t) = Q(0)e^{\gamma t}, \tag{15}$$

where $Q(0)$ is the present value at time 0 of the expected future costs of extraction, given that the cost of the mine which is producing at time t satisfies condition (18). Define $f(C) \equiv akC^{-(1+k)}$, the density function of extraction costs. Then $Q(0)$ may be written as

$$Q(0) = \int_0^\infty C(t)e^{(m-r)t}f[C(t)]\, dt. \tag{16}$$

Then, substituting for $f(\ )$ and for $C(t)$ from (9), and using (15) it is seen that

$$Q(0)^{1+k} = \left(\frac{\alpha}{\alpha + r - m}\right)^{-k} ak \int_0^\infty \exp\{\{m - r + k(m - \gamma)\}t\}\, dt, \tag{17}$$

or

$$Q(0) = \left[\left(\frac{\alpha}{\alpha + r - m}\right)^{-k} \frac{ak}{[k(\gamma - m) + r - m]}\right]^{1/(1+k)}. \tag{18}$$

[8] The fraction of mines producing in the interval $(t, t + dt)$ is $\{F[C(t + dt)] - F[C(t)]\}\{1 - F[C(t)]\}^{-1}$, where F is the distribution function of real costs. Taking limits, this may be written as $h[C(t)]C'(t)dt$, where $h(C) \equiv F'(C)/(1 - F(C))$ is the hazard rate and primes denote derivatives.

[9] Note that this is just the condition required for it to be costly to advance production.

[10] In the interest of tractability we are assuming that all mines are expected to produce eventually. Alternatively, $Q(0)$ could be determined by the assumption that all mines, if they are to produce, must do so by some final date T^*.

The cost of the mine which produces at time t in equilibrium is given by equation (9) where $Q(t)$ is defined by equations (15) and (18).

All mines with real costs less than $C(0)$ produce immediately at time 0. The remaining mines defer production until their cost satisfies condition (9). Thus in equilibrium the current cost of the currently producing mine rises at the rate γ. Each mine finds it worthwhile to postpone production until the cost of further postponement due to continuing as a member of the remaining pool of high cost mines exceeds the cost of advancing production. Since, under symmetric information, it would be optimal to postpone production indefinitely, the timing of production can be regarded as a costly "signal" of firm type. Then the model is one of dynamic costly signaling in which firms reveal their types sequentially. However, to treat the act of production as a costly signal is to strain usage, and the equilibrium is perhaps better described as one of sequential type revelation. In any case, we note that in this model the announcement of production is greeted with an immediate increase in the stock price, as the cost of production of the currently producing firm is a fraction $\mu \equiv \alpha/(\alpha + r - m) < 1$ of the present value of the costs of the average firm which is yet to produce.

Our results thus far rely on the somewhat ad hoc specification of the firm's objective function (1). It is instructive to consider how our results change if the firms pursue a more classical policy of (market) value maximization. Under a policy of value maximization, a mine with real cost C will produce if and only if its cost of extraction does not exceed $Q(s)$:

$$Ce^{ms} \leq Q(s). \tag{19}$$

To avoid technical difficulties, it is convenient to assume that each firm follows a policy of *virtual* continuous value maximization, so that a firm with initial cost $C(t)$ produces at time t where

$$C(t) = \mu Q(t)e^{-mt} \tag{20}$$

and $\mu < 1$. This policy approaches continuous value maximization as $\mu \to 1$.

Suppose that the probability distribution of mine costs is given as before by expression (12). Then, noting the similarity between expressions (9) and (20) which determine the optimal production policies under the two policies, it is apparent that the equilibrium under value maximization is identical to that derived above when $\alpha/(\alpha + r - m) = \mu$. In particular, the optimal production policy is given by (20), where

$$Q(0) = \mu^{-k}\left[\frac{ak}{k(\gamma - m) + (r - m)}\right]^{1/(1+k)}, \tag{21}$$

$$Q(t) = Q(0)e^{\gamma t}, \tag{22}$$

and $\gamma = (r - m\eta)/(1 - \eta)$, where $\eta = k(1 - \mu)$. Thus as $\mu \to 1$, $\gamma \to r$, and the current cost of the currently producing mine grows at a rate approaching the riskless interest rate. Moreover, since $C(t)e^{mt} \to Q(t)$ as $\mu \to 1$, as the policy

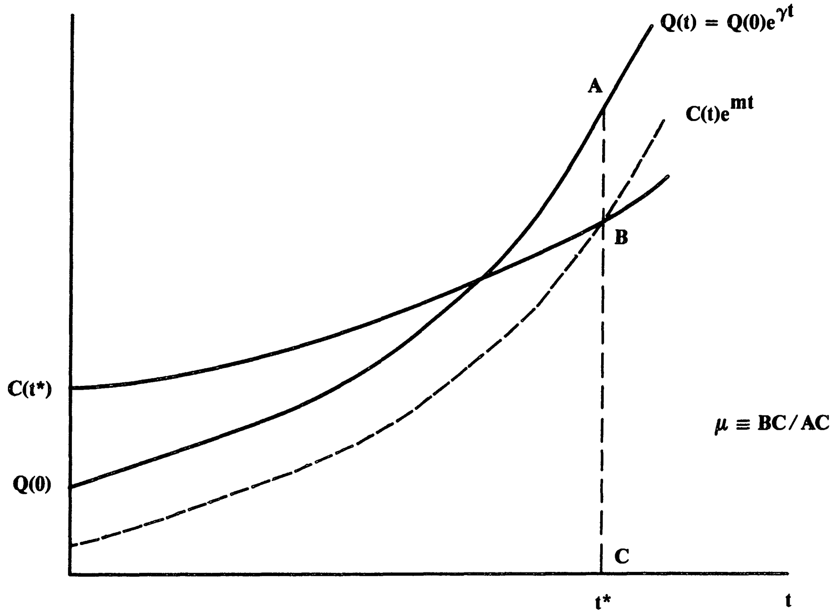

Figure 1. Equilibrium extraction policy for alternative objective functions. A mine with real cost $C(t^*)$ produces at t^*, where its cost of extraction is a fraction μ of the present value of the average costs of extraction of remaining undeveloped mines, $Q(t^*)$. Under objective function (1), $\mu \equiv \alpha/(\alpha + r - m)$; under value maximization, $\mu \to 1$.

approaches that of pure value maximization, the capital gain on the announcement of production approaches zero.[11]

Figure 1 illustrates the nature of the equilibrium under the alternative objective functions. The ratio of the distances $AB:AC$ is equal to $\mu \equiv \alpha/(\alpha + r - m)$. This represents the ratio of the average cost imputed by the market to unexploited mines to the cost of the currently producing mine.

The mines in our model are atomistic, each producing unit output. Real mines correspond to collections of these mines. The model thus predicts that within a given mine the low cost ore will tend to be exploited first, which corresponds with the conventional view, though for different reasons.

II. Generalization from the Gold Mine Example

We have shown that, even under a policy of value maximization the classical NPV rule need not emerge as the criterion of (private) investment optimality under conditions of information asymmetry. Once again we see how critical is the assumption of symmetric information for the simple decision rules yielded by the classical theory of corporate finance; unfortunately, the decision rules which emerge from informational considerations seem much harder to imple-

[11] Note that market value maximization in this case is not equivalent to full information value maximization because of the effect of the (non)-production decision on investor perceptions. For a similar point in a different context see Rotemberg and Scharfstein (1989).

ment,[12] and while our result has been derived under the assumption that the firm has no means of signaling its quality, it seems likely that a similar result will emerge so long as there is any residual uncertainty about the quality of the mine.

Our analysis yields in principle a test of whether or not firms follow a policy of value maximization. A non-zero price reaction to the announcement that a mine is to be put into production is inconsistent with value maximization, and if the alternative is the objective function (1) which is a generalization of the type usually found in signaling models,[13] then the price reaction may be used to estimate the discount factor, α.[14]

Although we have considered a gold mine for reasons of analytical tractability,[15] similar results follow in more complex settings, and the gold mine example is intended as a parable of a more general phenomenon of latent, or only partially observed, asset values. As we saw, the effect of latency was to induce managers to realize the asset value early by converting it into cash whose value is universally observable. Other authors have suggested the possibility of a similar outcome,[16] and the financial press contains frequent references to hidden or latent assets.[17]

McConnell and Muscarella (1985) report that stock prices react positively to the announcement of investment projects; this is consistent with the view that the value of the investment opportunities is known only imperfectly to the market. Our theory then suggests that if management places any weight on the current stock price in their objective function, then these opportunities will be exploited too early relative to the usual NPV criterion.

Similarly, Klein (1986) reports that announcements of asset sales are accompanied by positive stock price rections.[18] Again, if the current stock price is an element of the managerial objective function, it follows that asset sales are at least partially motivated by the desire to signal value, and that they will be made even when the sale price does not reflect the full value of the asset. In the model of the previous section, the lower is the discount factor α the more weight is placed on the current stock price and the earlier are the mines exploited. It follows that an exogenous shock that decreases α, such as a sudden threat of takeover, can be expected to trigger increased asset disposals that are costly to

[12] See Heinkel (1978), Miller and Rock (1985), and Constantinides and Grundy (1989) have also presented models in which informational considerations affect investment policy. Dividend policy, capital structure decisions, and optimal call strategies inter alia are all affected by similar considerations.

[13] See Miller and Rock, op. cit, and Harris and Raviv (1985) for example. For a criticism of these objective functions see Dybvig and Zender (1987).

[14] Farrow (1985) provides evidence that actual resource extraction policies are inconsistent with value maximization.

[15] This allowed us to ignore the effect of timing on the present value of the revenues.

[16] Stein (1988) has developed a model somewhat similar to ours, in which for certain parameter values, managers may advance the exploitation of a resource in order to forestall a takeover. Narayanan (1985) presents a model in which the manager chooses projects with short payback periods in order to signal his own ability.

[17] The *Wall Street Journal* (January 20, 1989) in an article on "Hidden Assets" in Japan reported that many U.S. companies were sitting on "potential gold mines," that some companies were moving to "reap value" from these assets, while some bankers wonder if "companies are moving too hastily to reap one time gains."

[18] Interestingly, the positive reaction is only for announcements in which the sale price is specified.

the firm. The takeover threat forces the incumbent management to prove the value of its assets, and the only way in which this can be accomplished is by sale.

It has been suggested that real estate holdings which have relatively low yields may be difficult for the market to reflect accurately in share prices.[19] Consider for example an industrial company that owns its own head office building, or a chain of supermarkets that owns its stores. Ownership will contribute to earnings only by the amount of the rent savings, and it may be difficult for investors to distinguish that part of earnings which is implicitly rental income and should be capitalized at a low rate from that which arises from industrial or grocery product sales and should be capitalized at a higher rate.[20] We might therefore expect those companies whose stock prices do not fully reflect the value of their real estate holdings to sell them and lease them back, even if it is costly to do so.[21] And we might expect a positive stock price reaction to the announcement of such transactions even though the transaction has a negative net present value for, as we argued above, the stock price will depend on whether the firm is continuously maximizing its stock price.

As a final example of a class of assets which may be hard for the market to value, consider the net pension assets of a company which depend on the value of the pension rights held by employees. Prior to the tax imposed by the Miscellaneous Revenue Act of 1988, terminations of defined benefit plans were not infrequent; Alderson and Chen (1986) find a positive stock price response to the announcement of plan termination; this is consistent with the values of pension assets being only partially known to the market and the management maximizing an objective function similar to expression (1). On the other hand, Vanderhei (1987) finds a positive stock price response for only part of his sample, and Bruner, Harrington, and Marshall (1987) find no abnormal returns. While the lack of a stock price response may be interpreted in terms of market efficiency and certain theories of pension property rights, it should be noted that it is consistent also with the hypothesis that investors do not possess detailed knowledge of individual plans, but that firm managements follow policies of market value maximization. This points to the care that must be taken in interpreting event studies when the event in question is a corporate action.[22]

Our analysis of the extraction policy of the gold mine has rested on the assumption that it was not possible for the firm to communicate information about its costs to investors. This is not an unreasonable assumption so long as the investors are small. However, it is not plausible for large investors who might buy up the whole mine. How is our analysis changed by the existence of large investors who can assess mine costs? If the large investors are individuals, then they do not have an incentive to buy up high cost mines that are just about to

[19] Hite et al. (1987) report that firm liquidations are frequently associated with management complaints of "undervaluation of natural resources and real estate holdings, assets that had appreciated in relation to their historical cost carrying value" (p. 233).

[20] We shall argue below that, depending on the convention, it may not be worthwhile for the individual investor to make such calculations.

[21] "Investors can profit from real estate ··· provided a company exploits the value of its holdings. It can do this by swelling, leveraging, or upgrading properties ···." Heard on the street. *Wall Street Journal*, January 29, 1989.

[22] A similar point is made by Acharya (1988) and by Eckbo, Maksimovic, and Williams (1989).

produce and to hold them out of production, thereby reducing the present value of the extraction costs. More generally, it seems that large investors or corporate insiders may have an advantage in operating latent assets, since such investors need not concern themselves with market values.[23] Note that this does not apply to large *corporate* investors who must themselves be concerned about market values unless they in turn are owned by large individual investors. Thus a firm will be worth more if it is controlled by a large investor who is concerned with intrinsic value as well as market value.[24] And second, a firm will be worth more if it is owned by the managers rather than publicly held, since it will then be free to pursue the goal of intrinsic value maximization. This accords well with the popular view that firms go private in order to free themselves from the pressures for short term performance that a stock market quotation engenders. Of course in practice there are riskbearing costs associated with management ownership and dominant shareholdings; therefore, these are not necessarily optimal ownership arrangements.

III. Equilibrium Information Collection in a Market of Small Investors

Let us turn now from the problem of a firm which owns latent assets to the incentives for individual investors to collect information about firm asset values when it is costly to do so. The main point I shall make in this section is that for long-lived assets the incentive for an individual investor to collect information may depend heavily upon whether other investors are collecting information and that, as a result, there may exist more than one equilibrium, in only one of which investors collect information; in the other equilibrium the assets remain latent.

Assume without loss of generality that each of N individual investors can collect information about the latent asset value of any one of M firms at a cost c, but that this information cannot be communicated credibly to other investors. Consider then an overlapping generations economy in which each generation lives for three, not necessarily equally spaced periods, and a new generation is born every second period at t, $t + 2$, $t + 4$, etc.[25] In the first period of life (period t) generation i ($i = t/2 + 1$) uses its labor income to purchase shares from the previous generation, which in turn uses the proceeds for consumption. Labor

[23] Of course, they must be concerned with market values when they come to sell, but there is a presumption that market value maximization will not be a continual concern to them as it is to the management of a firm owned by small investors. It is possible that a managerial policy of acquiring latent assets, rather than simply agency costs, accounts in part for the association between undistributed cash flow and premiums in going private transactions observed by Lehn and Poulson (1989).

[24] This complements the theory of Shleifer and Vishny (1986) which also predicts that firms will be worth more when there is a large shareholder.

[25] The intent of this assumption is to represent the short decision horizon of most investors. The effect of this is to make non-income yielding assets whose value cannot be realized by sale or borrowing and which cannot appear on a balance sheet for, say, 10 years essentially valueless. Even institutional investors with long apparent horizons may have short effective horizons because of agency problems which induce frequent monitoring of the investment manager's performance.

DeLong, Shleifer, Summers, and Waldmann (1989) also employ an overlapping generations assumption to develop a model in which market values depart from fundamentals. However, in their model the departure is caused by the existence of noise traders. Common, and essential to both models, is the market incompleteness captured by the overlapping generations assumption.

Table I

Overlapping Generations Model with Information Production

	Generation i			Generation $(i+1)$
Time	t	$t+1$	$t+2$	$t+3$
Firm Assets	$a_{i-1} + l_{i-1}$ $+l_i$	$a_i + l_i$	$a_i + l_i$ $+l_{i+1}$	$a_{i+1} + l_{i+1}$
Generation i	Receives labor income. Purchases shares from generation $(i - 1)$. Purchases information about l_i.	Receives information about l_i purchased at t. Trades.	Sells shares to $(i + 1)$. Consumes. Dies.	
Share Price: Reflects l_i?	No	Not at open. At close if $j > m$ investors purchased information.	If reflected at $(t + 1)$ or publicly revealed (probability:μ).	Yes
Share Price:				
At open[a]		$a_i - D(1+r)^2$		$a_{i+1} - D(1+r)^2$
At close:				
if l_i reflected		$a_i + l_i - D$	$a_i + (l_i - D)(1 + r)$	
if l_i not reflected		$a_i - D$	$a_i - D(1 + r)$	

[a] If different from closing price.

income in the first period of life may also be used to purchase information about the latent asset values of individual firms: this information becomes available at random times during trade in the second period. In the second period $(t + 1)$ individuals suffer random wealth shocks that induce them to trade; at this time individuals who purchased information in the first period may use it to trade as it becomes available. Individuals are assumed to be risk neutral, but to be restricted in the amounts they can purchase or sell short. The key assumption we shall make is that the market price of a firm does not reflect the private information of investors in the second period, until there is some minimum number of investors informed about the firm. In the third period $(t + 2)$ the current generation sells its shares to the new generation of individuals, consumes, and dies.

The information structure of the model is illustrated in Table I. In the first period of life of generation i, time t, a fraction f of firms are endowed with latent assets, $l_i/(1 + r)$, where $l_i = \pm\Pi$ with equal probability, and r is the riskless interest rate; latent assets yield no current return but appreciate in value at the riskless interest rate. At the beginning of $(t + 1)$, before trade occurs, the value of latent assets held by the firm in *generation* $i - 1$ is revealed if it is not already known to its investors. Hence the public assets of the firm (in the second period of life of generation i) at time $t + 1$, a_i[26] are equal to the sum of the public and latent assets from the previous generation: $a_i = a_{i-1} + l_{i-1}$. Public assets yield a

[26] All asset values are expressed on a per share basis.

certain return each period equal to the interest rate. Information about l_i becomes available at random times during trading at period $t + 1$ to all investors who paid the information acquisition cost c at time t.

For simplicity we shall assume that share prices react in a discontinuous fashion to the purchases and sales of informed investors. In particular, we assume that each informed investor is able to buy/sell a maximum of one share, and that the share price does not reflect the private information until m shares have been purchased/sold by the informed, at which point the price changes to reflect the private information[27] which is thereby revealed.

If the latent asset value l_i is not reflected in the share price as a result of trading at time $t + 1$, there is an (exogenous) probability, μ, that the value will be revealed prior to the opening of trading at $t + 2$; failing this, the latent asset value will certainly be revealed prior to the opening of trading at $t + 2$ (period 2 of generation $i + 1$). The important feature of the model for our purposes is that information about latent asset values may not be reflected in asset values until the next generation.

Note that uninformed investors lose on average by trading with the informed in period 2, because informed investors are more likely to be sellers of shares with latent liabilities (<0), and purchasers of shares with latent assets. Let the expected loss per share for the uninformed from trading with the informed in the second period of life be d, and denote by D the value of the perpetuity of these expected losses, $D \equiv d/R$ where $R \equiv [(1 + r)^2 - 1]$. Then it may be verified that the following is an equilibrium: The share price at the open of trade at $t + 1$ is given by $a_i - D(1 + r)^2$. The price at the close of trade at $t + 2$, if the latent asset value has not been revealed by trade, is $a_i - D$. If the latent asset value has been revealed, the price is $a_i + l_i - D$. Similarly, the price at $t + 2$ is $a_i - D(1 + r)$ if the latent asset value has not been revealed, and $a_i + l_i(1 + r) - D(1 + r)$ if it has. Finally, the price at the open of trade at $t + 3$ of the following generation is $a_{i+1} - D(1 + r)^2$.

An informed investor who is able to transact at the opening price at $t + 1$ will realize an immediate expected profit of $f\Pi$ if l_i is revealed in the course of trading. Given our assumption about price adjustment, if j, the number of investors who acquire information about a frim, exceeds m, the first m of them will realize an expected profit of $f\Pi$ when the price adjusts after the transaction of investor m, while the remaining $(j - m)$ will realize no profit since they will not be able to transact at the full information price. Therefore, if there are $j \geq m$ informed investors, the expected profit per investor from price adjustment in period 2 is $f\Pi(m/j)$. If $j < m$, there will be no price adjustment in period 2; however, informed investors may yet profit from asset value revelation in period $t + 2$. The present value of the expected profit (as of period 2) is $\mu f\Pi$ where μ is the previously defined exogenous probability of value revelation in period $t + 2$.

Define η_{jt} as the probability that exactly j investors in generation t become informed about a particular firm. Then V, the Period 2 present value of the expected reward to becoming informed about a particular firm, given that a

[27] The model can be thought of as a simplification of that of Diamond and Verrecchia (1987), which has the same implications for information collection in our intertemporal setting.

fraction f of firms have latent assets, may be written as

$$V = f\Pi[\textstyle\sum_{j\geq m} \eta_{jt}(m/j) + \mu \sum_{j<m} \eta_{jt}]. \tag{23}$$

The first term in the parentheses corresponds to the event that at least m investors are informed about the firm, so that its value is revealed in the course of trading at $(t + 1)$; the second term corresponds to the possibility of a public announcement of the latent asset value at $(t + 2)$. We consider a stationary environment in which M, the number of firms, and N, the number of individuals who decide to become informed, are time invariant. It is assumed that each individual acquires information about no more than one firm and that firm is chosen at random. Then the probability that a given firm is chosen by a particular investor is M^{-1}, and the distribution of the number of investors who become informed about any firm is binomial, where N is the number of trials, and M^{-1} is the probability of success on a given trial.

Under these assumptions V is a function of N, the number of investors who decide to become informed, and may be written as

$$V(N) = f\Pi\ [\textstyle\sum_{j=m}^{N} b_j(M/j) + \mu \sum_{j=1}^{m-1} b_j], \tag{24}$$

where $\eta_{jt} = b(N, j, M^{-1}) \equiv b_j$, the binomal probability of j successes in N trials.

Equilibrium in the market for information requires that all those who acquire information earn an expected return that compensates them for the cost of acquiring information, and that there be no incentive for further information acquisition: the equilibrium number of investors who acquire information, N^e, must satisfy

$$V(N^e) \geq \hat{c} \equiv c(1 + r), \qquad V(N^e + 1) < \hat{c}. \tag{25}$$

At low levels of N, and for small μ, $V(N)$ is increasing, because an increase in the number of informed investors increases the probability that the price will reflect that latent asset value. However, for larger N the probability of immediate price adjustment is already high, and further increases in N tend simply to increase the number of informed individuals over whom the reward of the price adjustment must be spread.

Figure 2 illustrates the two possible types of equilibria in the market for information. The $V(N)$ schedule is drawn for $\mu = 0$ and for $\mu > \hat{c}/f\pi$.[28] When $\mu = 0$, $V(N) = 0$ for $N < m$, because there is no possibility of price adjustment in the current generation; $V(N)$ increases at first for $N > m$ as the probability of price adjustment increases, but eventually turns down as the increase in the number of individuals among whom the fruits of price adjustment must be shared outweighs the increased probability of adjustment. $V(N)$ is a strictly increasing function of μ as shown, and for sufficiently large μ will be a monotone negative function of N.

It is helpful to distinguish two types of market according to the probability of exogenous information revelation. If $\mu > \hat{c}/f\Pi$, there is only one equilibrium in

[28] It is also possible that the $V(N)$ schedule is everywhere below $\hat{c}$. In that case it is never worth collecting information.

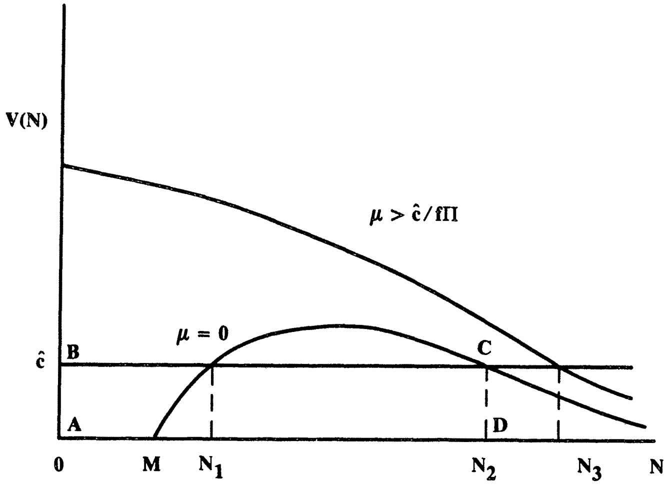

Figure 2. Equilibrium in the market for information acquisition.

the market for information at $N = N_3$. We might describe the security market in such an information gathering equilibrium as stochastically efficient since the probability that the period $(t + 1)$ closing price reflects the value of the firm's latent assets is $B(N_3, m; M^{-1})$ where $B(\ )$ is the probability of m or more successes in N trials. Note that as the cost of information acquisition approaches zero, $N_1 \to \infty$ and the probability that security prices fully reflect asset values approaches unity.

Consider the situation on the other hand when μ, the probability of exogenous information revelation, is low: $\mu < \hat{c}/f\Pi$. Now there exist two equilibria. $N = 0$ is an "informationless" equilibrium. In this equilibrium it is not worthwhile for anyone to gather information because no one else is gathering information and there is no way to profit from price adjustment. There is also an information gathering equilibrium at $N = N_2$; note that $N = N_1$ is not an equilibrium since at that point there is an incentive for additional investors to become informed.

The first lesson of this simple model is that there may be externalities to the collection of information in securities markets. For example, if oil company shares do not reflect the option value of reserves that can only be developed profitably at a higher oil price, there may be no incentive for individual analysts to estimate the value of these reserves and share prices may continue to ignore them. Nor will this problem be rectified by a single large investor (not included in our model) buying up the whole company unless this investor has an investment horizon which stretches over into the next generation.

While in this model we have made a simple distinction between assets whose values are publicly known and assets which are latent or observable only at a cost, it should be clear that almost all assets possess aspects of latency, and that the uncertainty surrounding corporate asset values can be progressively reduced as the cost incurred in investigation is increased. Our analysis suggests that it may not be worthwhile for an individual to invest heavily in information acquisition unless a sufficient number of other investors are also doing so.

Second, since there is more than one possible equilibrium, the particular equilibrium that prevails may be a matter of convention or of historical accident. For example, it has often been noted that the discounts from net asset value of closed end investment companies show considerable intertemporal variation. However, as Thompson (1978) has shown, the rewards from buying "underpriced" funds are quite modest, because prices do not adjust rapidly[29]—because not many investors attempt to arbitrage these securities. If, on the other hand, there were a well-agreed-on-formula for the appropriate discount based on expense ratios, investment performance, etc., then it is at least possible that there would exist an equilibrium in which it was worthwhile for investors to estimate the appropriate discount and adjustments to this would be rapid. The now abandoned proposal for Unbundled Securities Units may represent a case in which the bad equilibrium prevailed. Reputedly this innovation failed because of concern that the market for USUs would be insufficiently liquid, causing them to trade at a discount to the underlying securities. There is another equilibrium in which the USUs are regarded as perfect substitutes for the underlying security, thereby gain liquidity, and are priced as perfect substitutes.[30]

Third, note that in an information gathering equilibrium the number of individuals who collect information at time t is an increasing function of μ, the probability of exogenous information revelation at $t + 2$. The reason for this of course is that the greater the value of μ, the higher the probability that the asset will trade at its full information value during the life of the current generation, so that purchasers will realize the true full information value rather than handing the asset on to the following generation at a price which does not reflect this information. In a more general model, the higher the income stream and the longer the individual investor horizon, the higher is the proportion of the return realized by the investor that depends on fundamentals and the lower the proportion that depends on the terminal price; consequently, the greater is the incentive to collect information. Thus we should expect the prices of high payout assets to reflect the underlying asset values more precisely than those of low payout assets, ceteris paribus.

Consider next the determinants of D, the expected discount from asset value at which shares trade at the end of period $(t + 1)$. First, note that in an informationless equilibrium the discount will be zero, for then uninformed investors would experience no adverse selection in their period $(t + 2)$ trading. In the information gathering equilibrium informed investors exactly recoup their information costs by trading with the uninformed. Hence d, the expected trading losses of the uninformed per period per share, are equal to $(N_e/M)\hat{c}$ where N_e is the equilibrium number of informed investors. Hence the discount from asset value, D, is given by

$$D = \frac{N^e \hat{c}}{MR}, \tag{26}$$

where $R \equiv (1 + r)^2 - 1$, is the two period interest rate.[31]

[29] A similar point has been made by Summers (1986).

[30] Cornell and Shapiro (1989) present an interesting example of the failure of arbitrage in the U.S. Treasury bond market.

[31] See also Heinkel and Kraus (1987) for a model of differential returns to insiders and outsiders.

Equation (26) implies that the discount from asset value is proportional to the area of the rectangle *ABCD* in Figure 2. Since, from equation (24), the expected reward to information collection, V, is increasing in both $f\Pi$ and μ, it follows from inspection of Figure 2 that in an information gathering equilibrium D, the expected discount from asset value, is an increasing function of

(i) the uncertainty about latent asset value, as measured by $f\Pi$, since the greater is this uncertainty, the greater is the adverse selection problem faced by the uninformed. It follows that value maximizing firms have an incentive to reveal their values as accurately as possible.[32]

(ii) μ, the probability that the value of the latent assets will be revealed in period $t + 2$. Not surprisingly, information that is not likely to be revealed does not pose a threat to the uninformed.

It seems likely that information acquisition costs, c, would be higher for multidivisional conglomerate firms which are spread across unrelated industries. Although the effect of c on the discount is ambiguous, it is at least possible in this model that the diversified conglomerate pays a valuation penalty on account of the difficulty of evaluating its assets.

IV. Conventional Valuation

In the model of Section IV, share values may fail to reflect intrinsic values for two distinct reasons. First, the shares may be valued in an informationless equilibrium in which there is no incentive for individuals to collect information because an insufficient number of other individuals are collecting information. Second, in an informed equilibrium, the cost of information collection must be extracted from the uninformed: responding to their resulting losses from adverse selection, the uninformed will be willing to pay less than the expected value of the assets for the shares they purchase. There is also a third way in which prices may fail to reflect fundamental values, for the particular informed equilibrium that prevails may also be a matter of convention—the market may systematically employ the "wrong" model to value a particular class of securities, and yet there may be no natural forces tending to return the market to the "right" model.

To see this, assume now that the value of the latent asset, l is normally distributed with mean zero and variance, σ_1^2, and suppose that there exists a noisy signal of l:

$$s = l + \epsilon, \tag{27}$$

where $\epsilon = N(0, \sigma_\epsilon^2)$. s may be thought of as the valuation implied by an inappropriate model. Assume that the cost of observing l or the noisy signal is c, the same for both.

Then from equation (24) and the properties of the normal distribution it follows that $V^l(N)$, the expected reward of observing l when N investors observe l is

$$V^l(N) = f\sigma_1 \sqrt{\frac{2}{\pi}} \{\textstyle\sum_{j=m}^{N} b_j(m/j) + \mu \sum_{j=1}^{m-1} b_j\}. \tag{24'}$$

[32] Cf. Diamond (1985) and Ross (1989).

Consider next $V^s(\mathrm{N})$, the expected return from observing the noisy signal s when N investors also observe the same signal. If m investors acquire information about the same firm, the time period $(t + 1)$ price adjustment will be equal to $\hat{l} \equiv E[l \mid s]$, then the expected additional price change at time $(t + 2)$, conditional on information revelation and the signal will be zero. If no information is revealed in the course of trading at $(t + 1)$, the expected profit conditional on information revelation at $(t + 2)$ for an investor who received the signal s is $\mid E[l \mid s] \mid \equiv \mid \hat{l} \mid$. Thus $V^s(\mathrm{N})$ is given by

$$V^s(N) = f\sigma_1 \sqrt{\frac{2k}{\pi}} \{\textstyle\sum_{j=m}^{N} b_j (m/j) + \mu \sum_{j=1}^{m-1} b_j\}, \tag{28}$$

where $E[\mid \hat{l} \mid] = \sigma_1 \sqrt{\frac{2k}{\pi}}$ and $k = \sigma_1^2/(\sigma_1^2 + \sigma_\epsilon^2)$. Comparing (24′) and (28), it is seen that for all N $V^l(N) > V^s(N)$, so that given the number of investors who collect information, they would, not surprisingly, be better off collecting information about l rather than about s, the noisy signal of l. Despite this, we shall see that if N investors are collecting information about s, it may not pay any of them to switch to collection of information about l.

An individual who knows l will make an expected profit of $\mid E[\hat{l} \mid l] \mid$ in Period 2 if m or more investors collect information about s,[33] and, if the information is revealed at $(t + 2)$, will make a further profit whose expected value is $\mid E[l - \hat{l} \mid l] \mid$. If the price does not adjust at $(t + 1)$, then his or her profit at $(t + 2)$, conditional on information revelation, is $\mid l \mid$. Thus $\phi(N)$, the expected reward to collecting information about l when N investors collect information about s is given by

$$\phi(N) = f\{\textstyle\sum_{j=m}^{N} b_j[(m/j)E[\mid E[\hat{l} \mid l]\mid] + \mu E[\mid E[l - \hat{l} \mid l]\mid]] + \mu \sum_{j=1}^{m-1} b_j E[\mid l \mid]\}, \tag{29}$$

where b_j is the binomial probability that j investors collect information about s.

Using the properties of the normal distribution, this may be written as

$$\phi(N) = f\sigma_1 \sqrt{\frac{2}{\pi}} \{\textstyle\sum_{j=m}^{N} b_j[km/j + \mu(1 - k)] + \mu \sum_{j=1}^{m-1} b_j\}. \tag{30}$$

Thus, the incentive for an individual to collect information about s, or to use the "wrong" model, rather than collect information about l is proportional to

$$\textstyle\sum_{j=m}^{N} b_j[(m/j)(\sqrt{k} - k) - \mu(1 - k)] - \mu \sum_{j=1}^{m} b_j(1 - \sqrt{k}), \tag{31}$$

and this expression may clearly be positive for small μ—it is more likely to be positive the less noisy the signal (the larger is k).

The incentive to deviate from the equilibrium convention of collecting information on the noisy signal or imperfect model, s, is increasing in μ because the

[33] We are assuming that our deviant individual has no effect on price setting.

higher is μ the greater is the probability that the payoff received by the current generation will depend upon the underlying fundamentals rather than the noise signal s. Similarly, the more noisy is the signal, the greater is the incentive to deviate by collecting information on the underlying asset value, because the more noisy is the signal the greater is the reward from knowing the underlying asset value if it is revealed at $t + 2$; offsetting this is the consideration that the noisier is the signal, the less useful is knowledge of the underlying asset value for predicting the change in price during trading at $t + 1$; nevertheless, it can be shown that the former effect outweighs the latter.

Thus, we have shown that it may be advantageous for an investor to stick with the conventional valuation model even though that model is incorrect in the sense that there may be another model which predicts the asset payoffs better. The reason for this paradox of course is that the investor has only limited interest in the ultimate asset payoffs—he is also interested in the resale value of the asset, and that may depend on the conventional valuation model. Note that this result does not depend on the cost of information since we have assumed that the cost of both signals is the same.[34]

It is an interesting empirical issue whether asset market values are ever based on inferior valuation models. This will not be easy to determine, because for few assets is there a close relation between theoretical values and observable variables. Nevertheless, Modigliani and Cohn (1979) have suggested that the whole market was undervalued at times of high inflation because of mistaken conventions in treating real and nominal interest rates, and this hypothesis may be tested. As another example, consider gold again. Casual empiricism suggests that gold mines are valued by traditional discounting of expected net cash flows, although it is well known that the present value of an ounce of gold deliverable in the future is exactly equal to the current spot price. The question then is whether the difference between the value of gold computed at the current spot price and the value obtained by discounting projected sales revenues adds explanatory power to a regression in which the market value is the dependent variable and conventional brokerage house valuations are the independent variable.

V. Summary and Conclusion

We have considered the implications of costly information acquisition by investors from two perspectives. First, we have shown, using the parable of the gold mine, that if market prices do not reflect asset values and the managerial objective function places any weight on the current market price, then there will be an incentive to realize asset values early. We also showed that the market price reaction to the announcement of an asset realization will depend on the precise managerial objective function, and that in particular there will be no price reaction if the manager continuously maximizes the current stock price. If the cost of following such a policy is high, there will be a substantial advantage to

[34] The result is similar to sunspot theories of asset market equilibrium.

being owned by a single large investor or by management, since this will shift concern from the market price.

Second, we showed that in a market of small investors the rewards of information collection will depend in a non-monotonic fashion on the number of investors who collect information, since this will affect not only the frequency with which mispriced assets can be discovered, but also the speed at which the pricing discrepancies are eliminated and their discoverers rewarded. Thus it is possible to have equilibria in which assets remain latent despite relatively small information acquisition costs. Such situations create incentives for both managers and large investors who can gain control to convert assets to cash to demonstrate values. The externalities to the collection of information suggest that there may be advantages for small investors in delegating their information acquisition to a single entity; not only does this allow a sharing of costs, but also, by coordinating the investment decisions of the investors, ensures that the information acquired will be rapidly reflected in market prices to their mutual advantage.[35] It is possible that brokerage house analysts play such a role.

Appendix: An Equilibrium with Precommitment

Assume that the manager of the mine can precommit to an extraction policy and that the cost of extraction is not publicly observable. Then, if the interest rate is r, the real cost of extraction is C, and the rate of growth in the nominal cost of extraction is $m < r$, the intrinsic value of time 0 of a mine which is committed to producing at time τ is $g - Ce^{(m-r)\tau}$. Let $V(\tau)$ denote the market value at time 0 of a mine which precommits to producing at time τ, and assume that the manager of the mine maximizes a weighted average of the current market value of the mine and its intrinsic value:[36]

$$\max_{\tau} \alpha V(\tau) + (1-\alpha)[g - (e^{(m-r)\tau}]. \tag{A1}$$

The first and second order conditions for a maximum in this problem are

$$\alpha V' - (1-\alpha)(m-r)Ce^{(m-r)\tau} = 0, \tag{A2}$$

$$\alpha V'' - (1-\alpha(m-r)^2)Ce^{(m-r)\tau} < 0. \tag{A3}$$

Rational expectations require that the market value be equal to the intrinsic value:

$$V(\tau) = g - C(\tau)e^{(m-r)\tau}, \tag{A4}$$

where $C(\tau)$ is the real (time 0) cost of a mine that precommits to produce at time τ in equilibrium. Differentiating expression (A4) with respect to τ,

$$V' = -[C'(\tau) + (m-r)C(\tau)]e^{(m-r)\tau}. \tag{A5}$$

[35] Of course, to keep individual investors in the coalition, it will be necessary to randomize the order in which they receive the information. This differs from the randomization of messages by a monopoly information supplier in Admati and Pfleiderer (1986).

[36] This type of objective function which is virtually canonical in models of this kind contrasts with the market value maximization role considered in the body of the paper.

Substitution for V' from (A5) in condition (A2) yields, after some simplification:

$$\alpha C' = (r - m)C. \tag{A6}$$

Solving (A6) and imposing the condition that the lowest cost mine produces immediately, yields the equilibrium time to produce schedule:

$$\tau(c) = \left(\frac{\alpha}{r - m}\right)\ln(C/\underline{C}), \tag{A7}$$

where $\underline{C} \leq g$ is the real extraction cost of the lowest cost mine.

The equilibrium valuation schedule is obtained by solving (A7) for $C(\tau)$ and substituting in (A4):

$$V(\tau) = g - \underline{C}\ \exp\{(r - m)(1 - \alpha)\tau/\alpha\}. \tag{A8}$$

It can be shown that this schedule satisfies the second order condition (A3), and that (A8) is the unique valuation schedule to satisfy the Cho-Kreps (1987) Intuitive Criterion. Finally, we can calculate the market value assigned to a mine with initial cost C by subsituting for $\tau(C)$ from (A7) in (A8) to obtain

$$V(C) = g - \underline{C}^{\alpha}C^{1-\alpha}. \tag{A9}$$

As with the model without precommitment presented in the text, the information asymmetry induces early extraction, and it is the mines with the lowest cost that produce first. Thus the equilibrium in qualitatively similar to that of Section II.

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