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Decentralized Investment Management

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Introduction

THE MARKOWITZ (12, 13) paradigm for mean-variance portfolio analysis involves two main components: (1) the preferences (indifference curves in mean-variance space) of the beneficiary of the portfolio's performance and (2) the oportunity set (feasible region in mean-variance space) based on someone's subjective estimates of security expected returns, risks and correlations. Most discussions assume that two people are involved—the "client" (beneficiary) and the "investment manager." The former's preferences and the latter's judgements are somehow brought together and an optimal portfolio selected.

In practice the situation is often more complex. Some investment funds (e.g. mutual funds) have many beneficiaries. Others (e.g. some pension funds) are managed by more than one investment firm. The problem of optimal portfolio selection in such a context is considerably more difficult than in the traditional setting.

We will not deal with the problems associated with more than one beneficiary here. Instead we will focus on situations in which more than one investment manager is employed in the process of selecting an investment portfolio. What might explain the use of such procedures? What types of objective functions should such managers be given? How should funds be allocated among managers? Under what circumstances can myopic decision rules lead to overall results that are optimal? These are the questions we address.

Current Practice

If security markets are perfectly efficient and correspond to the original capital asset pricing model of Sharpe (18), Lintner (10) and Mossin (14), the solution to

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the investment management problem is straightforward: every client should invest in some combination of the "market portfolio" and a riskless asset. If markets are perfectly efficient but more complex (e.g. conform to a more complicated asset pricing model) the task is more difficult. Nonetheless, in such a world a single "consensus" set of predictions could be used for the management of a portfolio and such predictions would be relatively simple to obtain and use.

In fact, those in charge of large pools of money (e.g. corporate pension fund sponsors) act as if they do not believe that the market is perfectly efficient. They hire investment managers in the belief that the managers they select can make better predictions in at least some domains than can the average, or "consensus" investor.

Private pension funds now hold approximately $225 billion in assets; state and local retirement funds hold an additional $180 billion (25). Moreover, the typical large fund uses multiple investment managers. One survey (8) identified 18 funds, each of which used more than 15 managers. In 1980 the American Telephone and Telegraph Corporation employed 112 investment managers for its pension funds (8).

Another aspect of current practice seems puzzling: each manager of a multiply-managed portfolio is typically given an explicit or implicit objective function that is *myopic*—i.e. he or she is not expected to select securities that "fit well" with the remainder of the portfolio. Usually the manager does not even know the composition of the rest of the portfolio. In a recent survey of large pension funds only 23% of the respondents disagreed with the statement: "With multiple managers, the individual managers should not be held responsible for coordinating investments with the portfolios of other managers." (7).

How can this possibly be optimal? Portfolio theory holds that security covariances are crucial aspects of portfolio risk. Is it possible to ignore many of them and still obtain optimal results? If so, how? These questions are central in the analysis that follows.

Another aspect of current practice also appears to be contrary to first principles. Implicitly or explicitly clients often give their managers incentives to worry about *relative performance* as well as *absolute performance*. A manager might be told that a poor portfolio return is undesirable, but that a return below that of some reference such as Standard and Poor's 500-stock index is even worse.

There is much controversy about the actual efficiency of security markets. We will not deal with this issue. Rather, we take the phenomenon of decentralized investment management on its own ground. We assume that clients believe that they can find superior investment managers. Given that assumption, we then attempt to explain some of the key aspects of their behavior.

All the cases to be considered involve one "client" and one or more "managers." Moreover, we assume that the client has a well-defined objective which is a function of portfolio return and risk. While most pension fund sponsors behave as if this were the case, there are strong arguments for alternative approaches, as shown by Sharpe (20), Treynor, Priest and Regan (24), Tepper (21) and Black (3). Here too, we take the problem on its own ground.

Reasons for Decentralized Management

Investment managers perform many functions: trading, record-keeping, risk and return estimation and portfolio management. While special skills may exist in trading, record-keeping and management *per se*, we focus exclusively on the prediction function.

Rightly or wrongly, a client who employs several managers believes that the best predictions can be obtained by some appropriate blend of the predictions of the chosen managers. If so, why not have the managers provide their predictions, with the client blending them and performing the appropriate portfolio optimization? In other words, why not replace *decentralized management* with *decentralized prediction-making* and *centralized management*?

To answer this question one needs to consider the economics of the investment prediction industry. While the *average* cost of a set of predictions may be substantial, the direct *marginal* cost of providing them to another user is very small. Moreover, a good set of predictions should be worth more to a large investor than to a small one. More specifically—the value should be related to the amount of money that will be influenced by the predictions. The predictor has a (natural) monopoly on a set of predictions and, if desiring to maximize profit, will wish to employ price discrimination (or, to use a less perjorative term—value-based pricing).

For effective price discrimination resale must be precluded. The predictor will thus have an incentive to make it difficult for other managers to obtain his or her predictions. This is one reason why investment firms may not wish to provide their predictions to the client, since the latter might divulge them to competitors. But there is another, perhaps more important, reason. If several sets of predictions are used for a centralized decision it is difficult to determine the value of one set for the overall portfolio, reducing the scope for value-based pricing strategies.

Many managers refuse to "sell" their predictions. Instead they require clients to give them money to "manage." Their predictions are employed in the management of such money, with mangement fees based in whole or in part on the amount of money under management. Moreover, the total fee charged is almost always a monotonic increasing function of the amount of money managed, although the fee per dollar managed is typically a decreasing function of the money managed (another aspect consistent with the traditional model of price discrimination).

To represent such a situation we assume that the client cannot obtain a manager's predictions directly, nor infer them from the manager's investment decisions (as described by Sharpe (19) and Fisher (6)) and then use the inferred predictions to manage other money. We will, however, assume that the client knows what the best "blend" of the managers' predictions would be—i.e. has estimated a function relating the best prediction to the (unknown) predictions of the managers. This can be done via regression analysis of ex post results on ex ante predictions (using old data), as described by Ambachtsheer and Farrell (1) or judgementally. Both approaches are currently used.

Approaches to Decentralized Management

Why have more than one investment manager? In discussions of the practice, such as those of Ezra (5) and Williams (26) two themes dominate: diversification and specialization. For example, Williams (26) differentiates between "diversification of styles"—hiring experts on different sets of securities—and "pure diversification"—based on a desire "... to have more than one manager in case a manager makes a large error."

To make these concepts more rigorous, we will use the term *diversification of style* to cover a polar case in which each manager analyzes a completely different subset of the universe of securities. The term *diversification of judgement* will be used to cover the other polar case in which two or more managers analyze the same subset of securities. Of course most situations involve both aspects, but as will be seen, these can be treated as combinations of the two extreme cases.

Previous Research

Decentralized investment management has received little direct attention, but much of the theory of investment is, of course, relevant. Markowitz' (12, 13) mean-variance approach constitutes the basic structure to be used here. Although the assumption of unlimited risk-free borrowing and lending will not be invoked, much of the analysis relies on separation results similar to that obtained initially under this assumption by Tobin (22). Separation has been employed in other contexts by Lintner (11), Black (2), Brennan (4), Kraus and Litzenberger (9), and others. In part of the analysis we will utilize the idea of "consensus" forecasts; these could (but need not) be related to the key attributes of an efficient market model such as the original capital asset pricing model of Sharpe (18), Lintner (10) and Mossin (14), the zero-beta model of Lintner (11) and Black (2), the tax-related model of Brennan (4), the arbitrage pricing theory of Ross (16), or any other equilibrium model. For generality we simply refer to "consensus estimates," leaving the interpretation open.

The first direct approach to separation in a nearly-efficient market context was the active/passive dichotomy of Treynor and Black (23). Their paper relied on the single-index model of return generation, but the important results are easily generalized (as will be shown).

In a seminal paper on multiple investment management, Rosenberg (15) dealt with the situation we term "diversification of judgement." A later paper by Rudd and Rosenerg (17), while concentrating on other issues, covers some aspects of the situation we term "diversification of style."

Much of this paper involves the restatement, generalization, and extension of previous work. No claim is made for complete originality.

Assumptions

The overall objective to be maximized is:

$$u_p = e_p - v_p/t_p \tag{1}$$

where:

u_p: the "utility" of the portfolio

e_p: the expected return of the portfolio, using "optimum" estimates of security expected returns
v_p: the variance of the portfolio
t_p: the risk tolerance for the portfolio (the client's marginal rate of substitution of variance for expected return)

The client's risk tolerance (t_p) is assumed to be constant over the likely range of decisions regarding e_p and v_p.

Unrestricted short sales are assumed to be possible with no required impounds. We also assume that neither management fees nor trading costs are affected by the client's allocation of funds among managers nor by the objective functions assigned to the managers. In addition, we assume that it is possible to provide appropriate incentives to lead each manager to maximize the objective function specified by the client.

All expected returns and covariances are assumed to be stated on an after-tax basis for the client in question. And, as indicated earlier, restrictions are assumed to exist so that one manager's predictions may not be used by another manager or by the client for "in-house" money management.

A major (and extreme) assumption relates to predictions: we assume that all managers analyzing a set of securities agree about the risks and correlations of those securities (i.e. the entries in that portion of the overall covariance matrix). Moreover, we assume that the client can, if needed, obtain such consensus estimates of covariances. On the other hand, we assume that different managers may make estimates of security expected returns that differ from the "consensus" estimates (which may or may not be derived from an equilibrium model). In such a world, one hires managers for (hopefully) superior predictions of expected returns, not risks.

This assumption involves a troublesome lack of symmetry. Presumably two analysts with different information will make different estimates of both risk and return. Two defenses are offered for our approach. First, it conforms to at least some current practice: a number of investment managers subscribe to an econometric service that provides security risk estimates, adding their own estimates of expected returns and then determining the implied "optimal" portfolios. Another reason is related to the main question addressed here. Under this assumption myopic decision-making can in some circumstances lead to optimal results. Our approach allows for different information sets (obviously necessary to justify decentralized decisions) yet provides a setting in which such procedures can sometimes prove competitive with centralized decision-making. In the presence of significant disagreement about risks as well as returns, decentralized decision-making is likely to be significantly less desirable. Since we analyze the procedure in its best possible setting, the practical implications of our results should be assessed accordingly.

The First-Best Solution

Whilé (1) is the most natural form for the client's objective function, it is convenient to transform it from expected return equivalent units to variance equivalent units. Thus we assume that the objective is to maximize:

$$u_p = t_p e_p - v_p \tag{1 a}$$

subject only to the "full investment" constraint:

$$\textstyle\sum_{i=1}^{n} x_{ip} = 1 \tag{2}$$

where:

x_{ip}: the proportion of the portfolio invested in security i
n: the number of securities

To solve this problem one must maximize the Lagrangean function:

$$t_p e_p - v_p + z_p \left[1 - \textstyle\sum_{i=1}^{n} x_{ip}\right] \tag{3}$$

where:

z_p: a Lagrange multiplier for the portfolio (the marginal utility of money in variance equivalent terms)

The first-order conditions—which are both necessary and sufficient in this case—are:

$$\begin{bmatrix} 2c_{11} & \cdots & 2c_{1n} & 1 \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ 2c_{n1} & \cdots & 2c_{nn} & 1 \\ 1 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} x_{1p} \\ \cdot \\ \cdot \\ \cdot \\ x_{np} \\ z_p \end{bmatrix} = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix} + t_p \begin{bmatrix} e_{1p} \\ \cdot \\ \cdot \\ \cdot \\ e_{np} \\ 0 \end{bmatrix} \tag{3a}$$

where:

c_{ij}: the covariance between the returns of securities i and j
e_{ip}: the "optimum" estimate of the expected return of security i

This is more coveniently written in matrix form:

$$DY_p = K + t_p F_p \tag{3b}$$

where each upper-case letter represents the corresponding matrix or vector in (3a).

The solution to this system of equations is:

$$Yp = \underbrace{[GK]}_{\substack{\text{minimum-}\\\text{variance}\\\text{portfolio}}} + t_p \underbrace{[GF_p]}_{\substack{\text{divergences per}\\\text{unit of risk}\\\text{tolerance}}} \tag{4}$$

where:

$$G = \text{the inverse of } D$$

The vector Y_p represents the optimal portfolio (plus the associated value of z_p). Note that it can be considered to be the sum of two vectors. The first (GK) is a *portfolio* (in the sense that the values of the first n terms sum to 1), while the second is a constant (t_p) times a set of *divergences* (in the sense that the first n terms sum to 0). For a client with no tolerance for risk (GK) is the optimal portfolio—thus it must be the minimum-variance portfolio, as indicated. The second vector (GF_p) indicates the optimal divergence of each holding (from that of the minimum-variance portfolio) per unit of client risk tolerance.

This *separation* result is similar to that of Lintner (11) and Black (2); as in their "zero-beta" model, all clients can be served by mixtures of two actual portfolios—one based on $t_p = 0$, the other on an arbitrarily large value of t_p.

Active-Passive Management

We turn now to the first case in which a client wishes to choose a portfolio that is optimal for a specified "blend" of predictions. In this case the ingredients are the consensus forecasts of expected returns and those of a single "active" manager—i.e. one with beliefs that diverge in at least some respects from those of the consensus. The client is assumed to have determined an appropriate weight w_k such that the optimal etimate of a security's expected return is given by:

$$e_{ip} = e_{ic} + w_k \underbrace{[e_{ik} - e_{ic}]}_{\substack{\text{manager } k\text{'s "alpha"} \\ \text{estimate for security } i \\ (= \alpha_{ik})}} \tag{5}$$

where:

e_{ic}: the "consensus" estimate of the expected return of security i

w_k: the weight associated with manager k's estimate of expected return for a security in the equation for the optimum estimate of expected return when more than one manager's estimate is to be used

e_{ik}: manager k's estimate of the expected return of security i

α_{ik}: manager k's estimate of the alpha value for security i

As indicated, the client feels that the best estimate of a security's expected return is the consensus value plus a portion of manager k's "alpha" (defined as the difference between manager k's estimate and that of the consensus of investors).

Rewriting (5) gives:

$$e_{ip} = [l - w_k]e_{ic} + w_k e_{ik} \tag{5a}$$

and

$$e_{ip} = w_c e_{ic} + w_k e_{ik} \tag{5b}$$

where:

w_c: the weight associated with the consensus estimate of expected return for a security in the equation for the optimum estimate of expected return

The system of equations (5b) is more conveniently written in matrix notation:

$$F_p = w_c F_c + w_k F_k \tag{6}$$

where:

$$F_c = [e_{1c} \cdots e_{nc}\, 0]'$$

$$F_k = [e_{1k} \cdots e_{nk}\, 0]'$$

Note, from (5*a*) that the weights in (6) sum to 1.

Substituting (6) in (4) gives the solution for this case:

$$Y_p = [GK] + t_p\{G[w_c F_c + w_k F_k]\} \tag{7}$$

Rewriting:

$$Y_p = \underbrace{GK + t_p G F_c}_{\substack{\text{optimal passive} \\ \text{portfolio}}} + \underbrace{w_k t_p G[F_k - F_c]}_{\substack{\text{optimal active} \\ \text{divergences}}} \tag{7a}$$

This version of the solution shows that the optimal portfolio can be treated as composed of two portions: (1) an *optimal passive portfolio* for the client ($GK + t_p GF_c$) and a set of *optimal active divergences* ($w_k t_p G[F_k - F_c]$) due to the manager's private information. The greater the reliability of this information, the greater should be w_k and the divergences of holdings from optimal passive (consensus) amounts.

Another grouping of the terms in (7) is also of interest:

$$Y_p = \underbrace{w_c[GK + t_p GF_c]}_{\substack{\text{optimal passive} \\ \text{portfolio}}} + \underbrace{w_k[GK + t_p GF_k]}_{\substack{\text{optimal active} \\ \text{portfolio}}} \tag{7b}$$

This version shows that the portfolio can be broken into two parts—an optimal passive and an optimal active portfolio. The appropriate overall portfolio can be considered a combination of these components, with the weights dependent on the active manager's predictive ability.

This decomposition can also be used to determine an optimal decentralized investment policy. The client's investment funds can be allocated in accordance with predictive accuracy, with the portion w_c invested by a passive manager and the portion w_k invested by the active manager:

$$a_c = w_c$$

$$a_k = w_k$$

where:

a_c: the proportion of the portfolio managed passively
a_k: the proportion of the portfolio invested by manager k

Each manager can be instructed to maximize a myopic objective function (i.e. one requiring no information relating to the rest of the portfolio). The passive manager is told to maximize:

$$u_c = e_c - v_c/t_p$$

where

u_c: the "utility" to be maximized by the passive manager
e_c: the expected return of a passively-managed portfolio, using "consensus" estimates of security expected returns
v_c: the variance of the passively-managed portfolio

The active manager is told to maximize:

$$u_k = e_k - v_k/t_p$$

where

u_k: the "utility" to be maximized by manager k
e_k: the expected return of manager k's portfolio, using manager k's estimates of security expected returns
v_k: the variance of manager k's portfolio

This suffices, since $[GK + t_p GF_c]$ maximizes u_c and $[GK + t_p GF_k]$ maximizes u_k.

Note that the client's overall risk tolerance (t_p) is used in each case. Thus each manager is instructed to act as if his or her portion of the fund were the entire portfolio. In this situation decentralized management with myopic decision rules can give completely optimal results.

Relative Performance Objectives

The active/passive approach is completely feasible if w_k does not exceed 1. But what if a client wants a manager to be more aggressive than normal—i.e. assigns a w_k value greater than 1 to the manager's predictions? In such a case w_c in (7b) becomes negative, implying a short sale of the optimal passive portfolio in order to generate extra money for active management. As a practical matter, short sale of a passive portfolio is likely to be costly or precluded entirely. However, the desired result can be obtained in another way. In fact, the optimal portfolio can be achieved for any value of w_k without hiring a passive manager if the active manager is given an objective function that includes *relative performance.*

The decisions required to maximize portfolio expected return do not differ from those required to maximize the expected difference between the return on the portfolio and that of some standard portfolio. To represent "relative performance" as a separate objective thus requires modification of the treatment of risk. For our purposes the appropriate standard is the optimal passive portfolio for the client. A client "concerned about relative performance" is interested in the likely divergence of the portfolio's return from that of the standard portfolio. This can be reflected in an expanded objective function:

$$u_k = t_p e_k - v_k - s_k d_k \tag{8}$$

where

d_k: the variance of the difference between the return of manager k's portfolio and the return of the optimal passive portfolio

s_k: the weight attached to relative risk in manager k's objective function

In (8) d_k is the variance of the difference between the return on the portfolio and that of the specified "index fund." A positive value of s_k indicates that the client considers relative risk to be bad, other things equal. A negative value indicates that he or she considers it good, other things equal; more specifically, for a given absolute risk (v_k) the larger the relative risk (d_k) the better.

Note that (8) is a myopic decision rule. It relies on the composition of an optimal passive portfolio, but does not require information about any decisions made by the client or by other managers.

The solution to (8) is:

$$Y_k = \underbrace{[GK] + t_p[GF_c]}_{\text{optimal passive portfolio}} + \underbrace{\{1/[1 + s_k]\} t_p G[F_k - F_c]}_{\text{active divergences}} \tag{9}$$

The similarity to (7a) is obvious. Clearly there is a direct correspondence between s_k in (9) and w_k in (7a). By providing the manager with an objective function that includes relative performance, the client can induce the manager to choose a portfolio that is optimal, given the client's assessment of the latter's predictive ability. To induce behavior that is less aggressive than normal, relative risk is penalized. To induce behavior that is more aggressive than normal, relative risk is rewarded.

In practice most clients explicitly or implicitly consider relative risk undesirable (over and above its contribution to absolute risk). In the case of a single active manager this is consistent with a belief that the manager's predictions are poorer than the manager considers them to be. This may well be a healthy attitude.

Diversification of Style

We turn now to cases involving more than one active manager. To make the notation as simple as possible we will usually focus on situations involving only two managers. Extensions to cases with more than two managers are, however, straightforward.

We begin with the polar case in which each manager analyzes a completely separate subset of the universe of securities, with no concern for the remainder or for any interrelationships between securities in the domain analyzed and those in the remainder of the universe.

Portfolio Attributes

If the client's funds are allocated between two managers, with the portion a_1 given to the first manager and the portion a_2 ($= 1 - a_1$) given to the second, the expected return of the portfolio will be:

$$e_p = a_1 e_1 + a_2 e_2 \tag{10}$$

Note that the expected return of each of the active portfolios is assumed to equal the amount estimated by the manager of that portfolio.

The variance of the overall portfolio will be:

$$v_p = a_1^2 v_1 + a_2^2 v_2 + 2a_1 a_2 v_{12} \tag{11}$$

where

v_{kl}: the covariance between the returns of the portfolios of managers k and l

and its utility to the client will be:

$$u_p = a_1 e_1 + a_2 e_2 - [a_1^2 v_1 + a_2^2 v_2 + 2a_1 a_2 v_{12}]/t_p \tag{12}$$

The goal is, of course, to maximize (12).

Improvement Procedures

In general, the first-best solution cannot be obtained in one step with decentralized management since covariances between the returns of the active portfolios must be taken into account. However, an iterative process can be used. Two types of procedures can lead to an improvement in u_p. By alternating between them, an overall optimum can, in principle, be obtained.

One improvement procedure involves reallocation of the client's funds, on the assumption that the investment managers will not alter their relative holdings of securities:

given e_1, e_2, v_1, v_2, v_{12}

select a_1, a_2 to maximize u_p.

In effect, the portfolios are treated as mutual funds in which more or less may be invested. Unfortunately, even with this limiting assumption the procedure requires estimates of the expected returns of the component portfolios—values that are not easily estimated.

The second improvement procedure singles out one manager for attention, assuming that the allocation of funds and the compositions of other managers' portfolios will remain unchanged. The manager in question is asked to maximize a function that includes his or her portfolio's expected return and variance plus its covariance with the remainder of the overall portfolio. Rewriting (12):

$$u_p = a_1[e_1 - a_1 v_1/t_p - 2a_2 v_{12}/t_p] + [a_2 e_2 - a_2^2 v_2/t_p] \tag{13}$$

Thus manager 1 can be instructed to maximize:

$$u_1 = e_1 - v_1/t_1 - 2a_2 v_{12}/t_p \tag{14}$$

with:

$$t_1 = t_p/a_1 \tag{14a}$$

Or manager 2 can be instructed to maximize:

$$u_2 = e_2 - v_2/t_2 - 2a_1 v_{12}/t_p \tag{15}$$

with:

$$t_2 = t_p/a_2 \tag{15a}$$

Note that this type of procedure does not utilize a truly myopic decision rule. In general, the selected manager must utilize information that depends on the composition of the remainder of the portfolio.

Modification for a Multi-Factor Return Generating Model

An interesting special case arises when security covariances can be attributed to covariances among a set of *factors*. Assume that returns are generated by the following process:

$$\tilde{r}_i = \sum_{j=1}^{nf} b_{ij}\tilde{f}_j + \tilde{h}_i \tag{16}$$

with:

$$\text{covariance} \quad [\tilde{h}_i, \tilde{h}_j] = 0 \quad \text{for all} \quad i \neq j \tag{16a}$$

where:

$\tilde{r}_i$: the return on security i (a random variable)
b_{ij}: the sensitivity (beta) of security i to factor j
$\tilde{f}_j$: the value of factor j (a random variable)
$\tilde{h}_i$: security i's residual return (a random variable)
nf: the number of factors

Under these conditions the last term in (14) will be:

$$2[a_2/t_p]v_{12} = \underbrace{[\underbrace{X_1'B}_{\substack{\text{portfolio 1}\\ \text{betas}}}]\,[Q\underbrace{B'X_2}_{\substack{\text{portfolio 2}\\ \text{betas}}}]2a_2/t_p}_{\substack{\text{penalty/rewards for}\\ \text{portfolio 1 betas}}} \tag{17}$$

where:

$X_k = [x_{1k} \cdots x_{nk}]'$

x_{ik} : the proportion of manager k's portfolio invested in security i

$$B = \begin{bmatrix} b_{11} & \cdots & b_{1,nf} \\ \vdots & & \vdots \\ b_{n,1} & \cdots & b_{n,nf} \end{bmatrix}$$

b_{kj} : the sensitivity (beta) of manager k's portfolio to factor j

nf : the number of factors

$$Q = \begin{bmatrix} cf_{11} & \cdots & cf_{1,nf} \\ \vdots & & \vdots \\ cf_{nf,1} & \cdots & cf_{nf,nf} \end{bmatrix}$$

cf_{ij} : the covariance between factors i and j

As indicated, the magnitude of this term involves the beta values of manager 1's portfolio relative to the factors and a set of penalties or rewards—one for each beta value. The latter are related to the beta values of the remainder of the portfolio and the covariance structure of the factors (Q).

Letting:

P_{kj}: the penalty for the sensitivity of manager k's portfolio to factor j

the objective function to be given a selected manager can be written as:

$$u_k = e_k - v_k/t_k - \sum_{k=1}^{nf} b_{kj}P_{kj} \tag{18}$$

where

b_{kj}: the sensitivity (beta) of manager k's portfolio to factor j

In effect, the manager is instructed to regard certain beta values as bad ($P_{k,j} > 0$) and others as good ($P_{k,j} < 0$), with relevant magnitudes indicated. The manager need not be told the actual composition of the remainder of the portfolio, but the decision rule is still not truly myopic since the beta value penalties depend on the holdings in the remainder of the portfolio.

A particularly interesting case occurs when all the securities in a manager's domain have the same beta value with respect to the first factor, the same beta value with respect to the second factor, etc. Then the portfolio beta values will be unaffected by the manager's security choices and (18) can be simplified to:

$$u_k = e_k - v_k/t_k \tag{19}$$

In this (very special) case the appropriate decision rule for each manager is myopic, although the rule for allocating funds *among* managers still requires estimates of covariances among the subsets of securities (and their expected returns).

As a practical matter myopic decision making in the case of nonoverlapping domains is likely to be desirable when the beta values of securities within a domain relative to any "important" factor do not differ substantially. A factor's importance will depend on the sensitivities of the remainder of the portfolio to the factors and on the factor's covariances with other factors.

Diversification of Judgement

We turn now to the other polar case, in which every manager analyzes the same subset of the universe of securities. We assume, of course, that due to different information sets, the managers reach different conclusions regarding security expected returns. As before, much of the discussion will employ a two-manager case for convenience.

As in the previous analysis of active and passive management we assume that the client has determined an optimal relationship between the predictions of the managers and an unbiased estimate of expected return, given all their available information. We will analyze two possibilities although they by no means exhaust the list of interesting alternatives.

If the client believes that the best estimate of a security's expected return is bounded by the managers' estimates, the equation for optimal estimates of expected returns can be written as:

$$e_{ip} = w_1 e_{i1} + w_2 e_{i2} \tag{20}$$

with:

$$w_1 + w_2 = 1 \tag{20a}$$

Note that we assume that a specific manager's predictive ability is the same for every stock analyzed. Note also that no use is made here of consensus predictions.

The set of equations (20) can be written more succinctly in matrix notation:

$$F_p = w_1 F_1 + w_2 F_2 \tag{21}$$

The optimal portfolio can be found by substituting (21) in (4):

$$Y_p = [GK] + t_p\{G[w_1 F_1 + w_2 F_2]\} \tag{22}$$

Re-arranging terms gives:

$$Y_p = w_1 \underbrace{[GK + t_p G F_1]}_{\substack{\text{optimal portfolio}\\ \text{for manager 1}}} + w_2 \underbrace{[GK + t_p G F_2]}_{\substack{\text{optimal portfolio}\\ \text{for manager 2}}} \tag{22a}$$

The similarity to our analysis of active and passive management is not surprising. And, as in that case, decentralized investment management with myopic decision rules can easily provide the first-best solution. Funds are allocated in accordance with predictive abilities:

$$a_1 = w_1$$

$$a_2 = w_2$$

with the following objective functions given to the managers:

$$u_1 = e_1 - v_1/t_p$$

$$u_2 = e_2 - v_2/t_p$$

Note that each manager is instructed to act as if he or she were managing the entire portfolio (i.e. the client's overall risk tolerance (t_p) is used in the objective functions).

The procedure may not be feasible in some cases. For example, consider a situation in which the client believes that there is a relationship between each security's alpha value and the alpha values estimated by the managers:

$$\alpha_{ip} = w_1 \alpha_{i1} + w_2 \alpha_{i2} \tag{23}$$

where:

α_{ip}: the "optimal" estimate of the alpha value for security i

There is no obvious reason for the weights to sum to 1 in this case. For example, if each manager makes unbiased estimates given his or her information, each value of w_k would be 1 if *only* that manager's predictions were used (i.e. it would correspond to a coefficient in a *simple* linear regression of alpha on predicted alpha). But the parameters in (23) correspond to coefficients in a *multiple* regression. Only if the predictions were completely uncorrelated with each other would they equal the corresponding coefficients from a simple linear regression. If this were the case, and if each manager's predictions were unbiased (given his or her information set) the sum of the w_k's would equal the number of managers—an amount clearly greater than 1. More realistic cases would generally lead to smaller sums, but the amount could easily exceed 1.

To determine the optimal estimate of a security's expected return in this case one must incorporate consensus predictions:

$$e_{ip} = e_{ic} + \alpha_{ip} \tag{24}$$

Given the definition of alpha, this can be written as:

$$e_{ip} = e_{ic} + w_1[e_{i1} - e_{ic}] + w_2[e_{i2} - e_{ic}] \tag{24a}$$

or:

$$e_{ip} = w_c e_{ic} + w_1 e_{i1} + w_2 e_{i2} \tag{24b}$$

As shown, the expected return can be considered a weighted average of three estimates; moreover, in this case the weights will sum to 1:

$$w_c = 1 - w_1 - w_2 \tag{25a}$$

$$w_c + w_1 + w_2 = 1 \tag{25b}$$

In principle one could treat this as an expanded version of the case previously analyzed and apply similar procedures to obtain the solution. However, if the weights for the managers' alpha predictions sum to more than 1, w_c will be negative. With the previous procedure this would require short sale of the optimal passive portfolio to finance the allocation of more than the client's total fund to the active managers. However, there is an alternative. The approach suggested for changing the "aggressiveness" of a single active manager can be used: i.e. relative risk can be included in the managers' assigned objective functions.

While many alternative combinations are possible, an interesting one involves the allocation of funds in accordance with the managers' predictive abilities:

$$a_k = w_k / \sum_{k=1}^{m} w_k \tag{26}$$

where

m: the number of managers

Each manager is then assigned an objective function including both relative and absolute risk:

$$u_k = e_k - v_k/t_p - [s_k/t_p]d_k \tag{27}$$

The attitude toward relative risk is given by:

$$s_k = [1/\sum_{k=1}^{m} w_k] - 1 \tag{28}$$

Note that each manager is given the same objective function. Relative risk *per* *se* is penalized ($s_k > 0$) if the sum of the weights for the managers' predictive abilities is less than 1; it is rewarded ($s_k < 0$) if the sum is greater than 1.

In practice many clients using more than one manager appear to explicitly or implicitly consider relative risk undesirable (over and above its impact on absolute risk). This is consistent with a belief that the investment managers consider their predictions better than they really are (i.e. their alpha values are biased away from zero)—a belief that may be well-founded.

Diversification of Style and Judgement

Often the investment managers hired by a client analyze different but not completely different sets of securities. In such a situation there are elements of both diversification of style and diversification of judgement.

Absent clever partitioning of the domains analyzed by the managers into subsets of securities with common values in the cells of the off-diagonal blocks of the covariance matrix, decentralized investment management with truly myopic decision rules cannot produce the first-best solution directly in such a situation. However, the optimal solution can be approached iteratively, by combining the approaches we have discussed.

First, one should partition the universe of securities into different domains in such a way that within each domain a single optimal estimation rule applies—e.g. one of the form:

$$e_{ip} = \sum_{k=1}^{m} w_k e_{ik} \tag{29}$$

The procedures described for *diversification of style* can then be applied directly, with each of these domains treated as a single "manager."

The funds allocated to a given domain can be divided among the managers who analyze that subset of securities using the procedures described for *diversification of judgement*—i.e. in accordance with the managers' predictive abilities for securities within that domain. For such securities, each manager can be assigned an objective function that takes into account covariance with all the securities in the portfolio that are not in the domain (using relationships of the form (14) or (18)). Such decisions are myopic in one sense—they do not take into account holdings of securities in the domain chosen by other managers. But they are not fully myopic—directly or indirectly they should, in general, take into account relationships with the securities in the portfolio that are not in the domain.

Conclusions

It is difficult to provide a useful summary of this rather complex topic. To a person schooled in portfolio theory, decentralized investment management with myopic decision rules seems likely to be a decidedly second-best approach to the problem. Moreover, the emphasis on relative performance seems misplaced, since it is difficult to see why this attribute should enter the client's utility function. The message of this paper for those with such feelings is simply that such practices may be sensible after all.

On the other hand, such practices may not be sensible. Some people involved in decisions to hire several investment managers may not fully appreciate the problems associated with security covariances and the rather limited conditions under which decentralized management with fully myopic decision rules can give desirable results.

At the very least, our analysis emphasizes the need to distinguish between cases in which one hires several managers to analyze one subset of securities (*diversification of judgement*) and cases in which different managers are hired to analyze different sets of securities (*diversification of style*). As we have shown these are quite different situations in terms of the potential efficiency of decentralized management and the procedures that can help attain that efficiency.

There is, of course, much more to this problem. We have assumed away many important aspects of the principal-agent relationship(s). We have also disregarded possibilities for Pareto-optimal procedures for combining the agents' information sets for use in a centralized decision without making any of the agents (or the client) worse off. Finally, we have sidestepped the entire issue of market efficiency and the potential value of the agents' information sets (including their analytic procedures).

In short, we have clearly provided necessary and sufficient conditions for the traditional final sentence in such a paper: More research on this subject is needed.

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